INFLUENCE OF INTENSE MASS-TRANSFER PROCESSES ON THE INTEGRAL CHARACTERISTICS OF AN ELECTRODYNAMICALLY ACCELERATED PLASMA

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From the Boltzmann–Vlasov equations for a partially ionized plasma a set of ordinary differential equations is derived to describe the electrodynamic acceleration of a plasma in the integral approximation. The influence of intense mass-transfer processes and drag forces on the electrodynamic acceleration of a plasma is analyzed.

Intense mass-transfer processes exert a pronounced influence on the characteristics of a moving plasma. The most complete description of the electrodynamic acceleration of a plasma is based on the physical kinetic equations. The Boltzmann–Vlasov equations for a partially ionized plasma in external and self-consistent electromagnetic fields have the form

$$\frac{df_\alpha}{dt} + v_i^\alpha \frac{df_\alpha}{dx_i} + \frac{e_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) \frac{\partial f_\alpha}{\partial v_i^\alpha} = J_\alpha + K_\alpha, \tag{1}$$

$$\operatorname{rot} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\operatorname{rot} \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}, \tag{3}$$

$$\operatorname{div} \mathbf{B} = 0, \tag{4}$$

$$\operatorname{div} \mathbf{D} = q. \tag{5}$$

Ordinarily the plasma is regarded as a particle system of identical electrons, ions, and neutral molecules. The Boltzmann–Vlasov equations then decompose into three equations for the electronic, ionic, and neutral components, and the plasma is said to be a three-fluid body.

The difficulties inherent in this description are obvious and so far have remained almost intractable. However, if we are interested in the fine structure of the processes involved and confine them to a more approximative description, we need merely take account of the influence of various factors and effects on the electrodynamic acceleration of a plasma.

We obtain the following three-fluid plasma-dynamic equations from the kinetic equations by the method of moments:

$$m_n \frac{\partial n_n}{\partial t} + \operatorname{div} m_n n_n \mathbf{V}_n = S_n, \tag{6}$$

$$m_e \frac{\partial n_e}{\partial t} + \operatorname{div} m_e n_e \mathbf{V}_e = S_e, \tag{7}$$

$$m_i \frac{\partial n_i}{\partial t} + \operatorname{div} m_i n_i \mathbf{V}_i = S_i. \tag{8}$$

The second moments of the kinetic equations give the equations of motion (conservation of momentum), which, neglecting gravitational forces, we write in the form

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$$m_n n_n \frac{d\mathbf{V}_n}{dt} = -\operatorname{grad} P_n + \frac{m_e n_e}{\tau_{en}} (\mathbf{V}_e - \mathbf{V}_n) + \frac{m_i n_i}{\tau_{in}} (\mathbf{V}_i - \mathbf{V}_n), \tag{9}$$

$$m_e n_e \frac{d\mathbf{V}_e}{dt} = -\operatorname{grad} P_e - n_e |e| [\mathbf{E} + \mathbf{j} \times \mathbf{B}]$$

$$- \frac{n_e m_e}{\tau_{en}} (\mathbf{V}_e - \mathbf{V}_n) - \frac{n_e m_e}{\tau_{ei}} (\mathbf{V}_e - \mathbf{V}_i), \tag{10}$$

$$m_i n_i \frac{d\mathbf{V}_i}{dt} = -\operatorname{grad} P_i + n_i |e| [\mathbf{E} + \mathbf{j} \times \mathbf{B}]$$

$$- \frac{n_i m_i}{\tau_{in}} (\mathbf{V}_i - \mathbf{V}_n) + \frac{m_e n_i}{\tau_{ei}} (\mathbf{V}_e - \mathbf{V}_i). \tag{11}$$

The third moments give the energy equations, which we shall disregard.

The last terms in Eqs. (9)-(11) containing the relaxation times τ_{en}, τ_{ei}, and τ_{in} are determined by inelastic collisions of the components (integral K_α). The set of equations (6)-(11), augmented with the equations for the energy and state of the gas, kinetic equations (in the form S_n, S_e, and S_i), and the Maxwell equations (2)-(5), forms the complete set of equations of three-fluid plasma dynamics.

The following condition holds for a quasi-neutral plasma:

$$n_e = n_i = n. \tag{12}$$

Introducing the center-of-mass velocity of the three fluids:

$$\mathbf{V}_{\mathrm{cm}} = \frac{n_i m_i \mathbf{V}_i + n_e m_e \mathbf{V}_e + n_n m_n \mathbf{V}_n}{m_i n_i + m_e n_e + m_n n_n}, \tag{13}$$

the mass density:

$$\rho = n (m_i + m_e) + m_n n_n, \tag{14}$$

the current density:

$$\mathbf{j} = |e| n (\mathbf{V}_i - \mathbf{V}_e), \tag{15}$$

and

$$P = P_n + P_e + P_i, \tag{16}$$

we obtain the set of equations of one-fluid magnetoplasma dynamics

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{V}_{\mathrm{cm}} = S, \tag{17}$$

$$\rho \frac{d\mathbf{V}_{\mathrm{cm}}}{dt} = -\operatorname{grad} P + e\mathbf{E} + \mathbf{j} \times \mathbf{B} + \mathbf{F}. \tag{18}$$

The equation of state of the one-fluid plasma is written in the customary form

$$P = R\rho T. \tag{19}$$

In the transition from three-fluid magnetohydrodynamics to the one-fluid case we obtain a generalized Ohm's law in the form

$$\mathbf{j} - \frac{\omega_e \tau_e}{B} \mathbf{j} \times \mathbf{B} + \tau_e \frac{\partial \mathbf{j}}{\partial t} = \sigma (\mathbf{E} + \mathbf{V}_{\mathrm{cm}} \times \mathbf{B}) + \frac{\sigma}{|e| n} \left(\operatorname{grad} P_e - \frac{m_e}{m_i} \operatorname{grad} P_i \right) + \sigma_\beta (\mathbf{V}_{\mathrm{cm}} - \mathbf{V}_n), \tag{20}$$

in which

$\omega_e \tau_e$ is the Hall parameter; (21)

$$\frac{1}{\tau_e} = \frac{1}{\tau_{ei}} + \frac{1}{\tau_{en}} + \frac{m_e}{m_i} \frac{1}{\tau_{in}}; \tag{22}$$

$$\beta = \frac{m_e}{|e|} \left(\frac{1}{\tau_{en}} - \frac{1}{\tau_{in}} \right); \tag{23}$$

$$\frac{1}{\sigma}=\frac{m_e}{e^2 n}\left(\frac{1}{\tau_{ei}}+\frac{1}{\tau_{en}}+\frac{m_e}{m_i}\frac{1}{\tau_{in}}\right); \tag{24}$$

$$\omega_e=\frac{|e|B}{m_e}. \tag{25}$$

We now introduce the integral characteristics of the moving plasma: its average mass m and the velocity v of the center of inertia of the plasma for mechanical processes; and the voltage and current in the circuit for electrical processes. We replace the magnetohydrodynamic equations by the Newton equations with variable mass, taking account of mass-change kinetic processes, and replace the Maxwell equations by the Kirchhoff equations for the electrical circuit, as is always admissible when the displacement currents in the accelerated plasma are small. Then as a result of this transformation the set of equations for the electrodynamic acceleration of a plasma is written in the form of a set of ordinary differential equations:

$$\frac{dm\mathbf{v}}{dt}=\frac{1}{2}I^2\frac{dL}{dz}-\mathbf{F}, \tag{26}$$

$$\frac{dm}{dt}=M(t,\ v,\ I,\ z), \tag{27}$$

$$\frac{dV}{dt}=-\frac{1}{C_0}I, \tag{28}$$

$$\frac{dLI}{dt}+RI+V_{\rm pl}+\frac{1}{C_0}\int I dt=0, \tag{29}$$

$$\frac{dz}{dt}=v. \tag{30}$$

The system (26)-(30) can be derived by the variational method, starting with the Lagrangian of an electromechanical circuit with moving meshes [4].

We now consider the influence of mass-transfer processes generated by the diffusion and recombination of particles of the accelerated plasma and its mass increase due to electrode ablation, charge transfer, and the sticking of electrons to ions on the electrodynamic acceleration of a plasma, and elucidate the relative contribution of friction and drag forces working against acceleration of the plasma. Taking all of these processes into account, we write the set of equations (26)-(30) in the form [1]

$$\frac{dm\mathbf{v}}{dt}=\frac{b}{2}I^2-(b_1\mathbf{v}+b_2\mathbf{v}m+b_3|I|\mathbf{v}+b_4v^2+\ldots), \tag{31}$$

$$\frac{dm}{dt}=-a_1m-a_2m^2+a_3|I|+a_4I^2+\ldots, \tag{32}$$

$$I=-C_0\frac{dV}{dt}, \tag{33}$$

$$\frac{dLI}{dt}+RI-V=0, \tag{34}$$

$$L=L_0+bz, \tag{35}$$

where a_i (i = 1, 2, 3, 4) and b_j (j = 1, 2, 3, 4) are proportionality factors affecting the electrodynamic acceleration process; they have already been discussed in [1].

For the solution of the set of equations (31)-(35) with the given values of the system parameters we specify the initial conditions:

$$z=0,\quad v=0,\quad I=0,\quad V=V_0,\quad m=m_0 \tag{36}$$

at t = 0.

In dimensionless variables the set of equations (31)-(35), reduced to canonical form, appears as follows:

$$\frac{dy}{d\tau}=y', \tag{37}$$

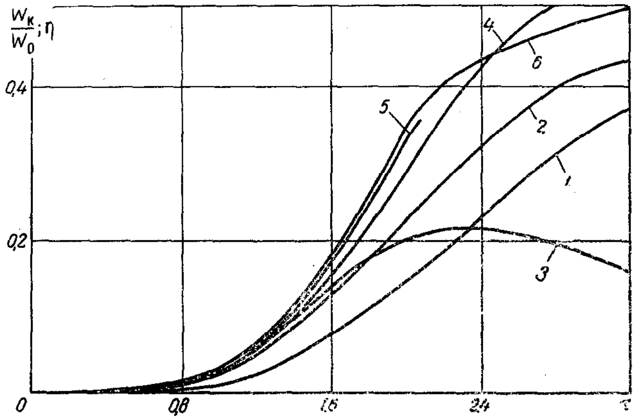

Fig. 1. Efficiency η and kinetic energy W_k/W_0 versus time τ for the parameters (45) of the set of equations (37-(40).

$$\frac{dy'}{d\tau}=\frac{q}{\mu}\varphi'^2-\frac{y'}{\mu}(\delta_1+\delta_2\mu+\delta_3|\varphi'|+\delta_4 y')-\frac{y'}{\mu}(-\gamma_1\mu-\gamma_2\mu^2+\gamma_3|\varphi'|+\gamma_4\varphi'^2), \tag{38}$$

$$\frac{d\varphi}{d\tau}=-\varphi', \tag{39}$$

$$\frac{d\varphi'}{d\tau}=\frac{\varphi-\alpha\varphi'-y'\varphi'}{1+y}, \tag{40}$$

$$\frac{d\mu}{d\tau}=-\gamma_1\mu-\gamma_2\mu^2+\gamma_3|\varphi'|+\gamma_4\varphi'^2, \tag{41}$$

where

$$\left\{\begin{array}{ll} q=\dfrac{b^2C_0^2V_0^2}{2m_0L_0}; & \alpha=R\sqrt{\dfrac{C_0}{L_0}};\\ \gamma_1=a_1\sqrt{L_0C_0}; & \gamma_2=a_2m_0\sqrt{L_0C_0};\\ \gamma_3=\dfrac{a_3C_0V_0}{m_0}; & \gamma_4=\dfrac{C_0^2V_0^2}{m_0\sqrt{L_0C_0}}a_4;\\ \delta_1=\dfrac{b_1}{m_0}\sqrt{L_0C_0}; & \delta_2=b_2\sqrt{L_0C_0};\\ \delta_3=\dfrac{1}{2}\dfrac{m_i}{e}\dfrac{C_0V_0}{m_0}; & \delta_4=\dfrac{b_4L_0}{bm_0};\\ \tau=\dfrac{t}{\sqrt{L_0C_0}}; & y=\dfrac{b}{L_0}z;\\ y'=b\sqrt{\dfrac{C_0}{L_0}}v; & \varphi=\dfrac{V}{V_0};\\ \varphi'=\sqrt{\dfrac{L_0}{C_0}}\dfrac{I}{V_0}; & \mu=\dfrac{m}{m_0}. \end{array}\right. \tag{42}$$

The initial conditions (36) in dimensionless variables for $\tau = 0$ assume the form

$$y=0,\quad y'=0,\quad \varphi'=0,\quad \varphi=1,\quad \mu=1. \tag{43}$$

The physical interpretation and values of the parameters q, α, δ_i, and γ_i, as well as the influence of the given processes on the integral characteristics of the moving plasma, i.e., the center-of-mass velocity y' of the plasma, the path y traversed by the plasma, the mass μ of the accelerated plasma, the voltage φ, and the current φ', have already been analyzed in [1].

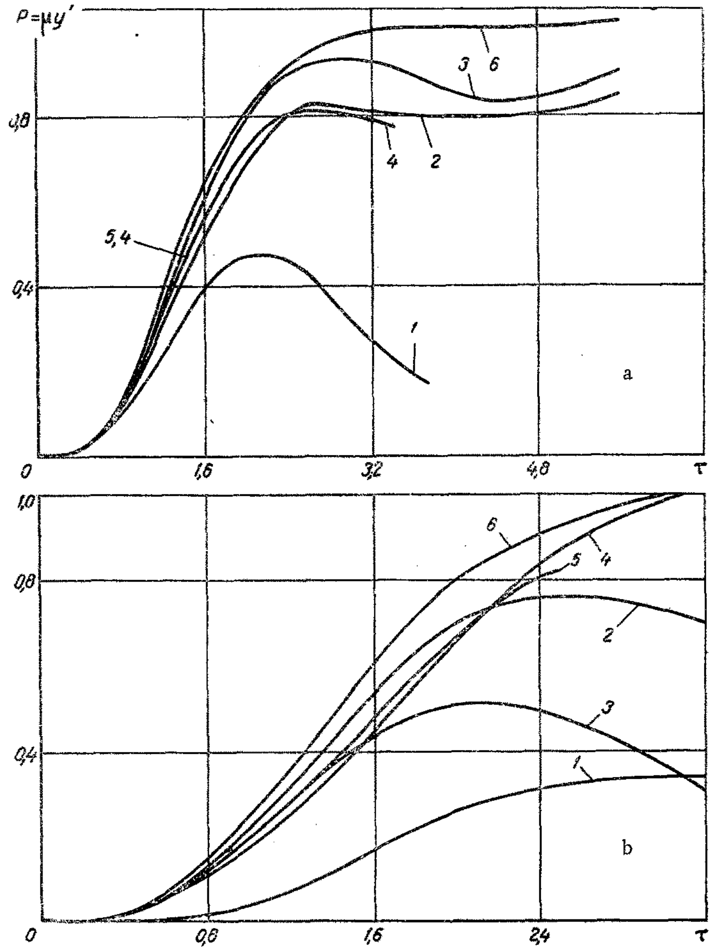

Fig. 2. Momentum $P = \mu y'$ versus time τ for the parameters (46) (a) and (45) (b) of the set of equations (37)-(40).

We now investigate the efficiency and momentum associated with the electrodynamic acceleration of a plasma, taking account of the given transfer processes and the friction and drag forces. The initial set of equations (31)-(35) has a first integral expressing the conservation of energy, and we write it in the form of an energy balance equation, both sides of which are divided beforehand by the energy W_0 stored in the capacitor [2]:

$$1 = \dot{\varphi}^2 + (1 + y)\varphi'^2 + 2\alpha \int_0^\tau \varphi'^2 d\tau + \frac{\mu y'^2}{2q} + \frac{W_L}{W_0}. \tag{44}$$

The set of equations (37)-(40), (44), subject to the initial conditions (43), has been solved numerically by the Runge–Kutta method with integration step h = 0.2 on τ and variable parameters δ_1, δ_2, δ_3, δ_4, γ_1, γ_2, γ_3, γ_4, and q. The results of the calculations are given in Figs. 1 and 2. To facilitate comparison the variation of like variables in the acceleration process is shown in each figure. The curves of Figs. 1 and 2b correspond to the following values of the parameters of the set of equations (37)-(40):

$$\begin{array}{lllll}
1 & q=0.24 & \alpha=0.1, & \gamma_1=\gamma_2=\gamma_3=\gamma_4=1, & \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1; \\
2 & q=1, & \alpha=0.1, & \gamma_1=\gamma_3=\gamma_4=1,\ \gamma_2=0.1, & \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1; \\
3 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=\gamma_3=\gamma_4=1, & \delta_1=\delta_2=\delta_3=\delta_4=0.1; \\
4 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=\gamma_3=\gamma_4=1, & \delta_1=\delta_2=\delta_3=\delta_4=0; \\
5 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=\gamma_3=\gamma_4=1, & \delta_1=\delta_2=0.1,\ \delta_3=\delta_4=0,
\end{array} \tag{45}$$

and the curves of Fig. 2a correspond to the values of the parameters

$$
\begin{array}{llll}
1 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=0,\ \gamma_3=\gamma_4=1,\ \delta_1=\delta_3=\delta_4=0,\ \delta_2=1; \\
2 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=0,\ \gamma_3=\gamma_4=1,\ \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1; \\
3 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=0,\ \gamma_3=1,\ \gamma_4=0.1,\ \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1; \\
4 & q=1, & \alpha=0.1, & \gamma_1=\gamma_2=0,\ \gamma_3=\gamma_4=0.1,\ \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1; \\
5 & q=1, & \alpha=0.1, & \gamma_1=\gamma_3=\gamma_4=1,\ \gamma_2=0,\ \delta_1=\delta_3=\delta_4=0,\ \delta_2=0.1.
\end{array}
\tag{46}
$$

Curves 6 in all the figures were plotted from the data of [3] for the case in which mass release does not take place in the acceleration process.

Figure 1 shows the variation with the time τ of the efficiency η, which is defined as the ratio of the kinetic energy of the plasma jet to the electrostatic energy stored in the capacitor and expended for acceleration of the plasma. This definition is expressed as follows:

$$\eta = \frac{W_{\text{к}}}{W_0} = \frac{\mu y'^2}{2q}. \tag{47}$$

The combined influence of the intense diffusion of particles, their recombination, ablation, and viscous forces on η can be estimated from curve 1. A tenfold reduction of the parameter γ_2 characterizing the recombination process or of the parameter δ_2, which accounts for the influence of diffusion viscous-friction forces, elicits an increase in the efficiency. The complete solution of the set of equations (37)-(40), (44) with the given parameters is represented by curve 3. It is clear that when the combined influence of all the indicated processes is taken into account the efficiency is sharply reduced, its maximum falling in the time interval τ = 2.2 to 2.6 and not exceeding 0.2150. Curve 4 in comparison with curve 3 enables us to assess the influence of drag and friction forces on the efficiency and kinetic energy. This curve corresponds to the case $\gamma_i = 1$, $\delta_i = 0$, and curve 5 to the case $\gamma_i = 1$; of the drag forces, only the forces associated with diffusion transfer friction and friction at the electrodes are taken into account.

An analysis of these curves shows that a large part of the energy is spent in overcoming the latter forces. Their reduction leads to a perceptible increase in the value of η. The solution of the set of equations (37)-(40), (47) for the parameters (46) is given in [2].

Graphs of the momentum P as a function of the time τ are shown in Fig. 2a; the momentum is defined as the product of the mass μ and velocity y':

$$P = \mu y'. \tag{48}$$

It is apparent from Fig. 2a that when mass-transfer processes are taken into account the resulting values of the momentum are smaller. The first maxima in these cases coincide in terms of the acceleration time τ, as in the case of η [1], and their values are between the limits from 0.4852 to 1.003. As remarked in [1], a reduction in the mass release naturally increases the plasma velocity, but the total momentum can decrease in this event.

The results of calculations based on (48) for the parameters (45) of the set of equations (37)-(40) are shown in Fig. 2b. As in the case of η (Fig. 1), when the combined influence of all the given physical effects resulting in mass transfer is taken into account, the developed momenta are reduced (curve 3). Comparing curves 1 and 2, we notice that increasing q in the cases indicated leads to an increase in the momentum.

The foregoing analysis shows that the enumerated mass-transfer processes, along with the friction and drag forces, produce a significant degradation of the resulting integral characteristics of the moving plasma in electrodynamic acceleration. The calculations performed above suggest ways in which the relative influence of these processes can be evaluated and improved.

NOTATION

f_α	is the distribution of each plasma component;
v_i^α	is the kinetic velocity of particles of the given component;
e_α	is the charge of the given particle species;
m_α	is the mass of the latter;
t	is the time;
x_i	is the i-th coordinate;
E, H	are the electric and magnetic field strengths;

$\mathbf{B}$	is the magnetic induction;
J_α, K_α	are the elastic and inelastic collision integrals;
$\mathbf{j}$	is the current density in plasma;
q	is the free-charge density in plasma;
$\mathbf{D}$	is the electric induction;
S_n, S_e, S_i	are the source functions describing the transition of one plasma component to another;
n_n, n_e, n_i	are the densities of neutrals, electrons, and ions;
m_n, m_e, m_i	are the masses of the latter;
$\mathbf{V}_n$, $\mathbf{V}_e$, $\mathbf{V}_i$	are the hydrodynamic velocities of the components;
τ_{en}, τ_{ei}	are the collision times of electrons with neutrals and ions;
τ_{in}	is the collision time of an ion with a neutral;
S	is the density of external mass sources;
$\mathbf{F}$	is the total plasma friction force;
T	is the absolute temperature;
R	is the gas constant of the plasma;
P	is the total pressure or momentum;
ω_e	is the Larmor frequency of electrons;
σ	is the electrical conductivity;
m	is the mass of accelerated plasma;
v	is the center-of-inertia velocity of accelerated plasma;
M	is a characteristic function describing the mass-change kinetics;
I	is the current;
V	is the voltage;
$b = dL/dz$	is the distributed inductance per unit length of accelerator;
C_0	is the capacitance;
L	is the overall system inductance;
R	is the circuit resistance;
V_{pl}	is the voltage drop across plasma;
z	is the coordinate of plasma center of inertia;
m_0	is the initial mass of plasma before acceleration;
L_0	is the inductance of input leads and capacitor;
W_L	is the energy spent in mass-transfer processes and other losses;
W_0	is the stored energy in capacitor.

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